

III Semester B.A./B.Sc. Examination, November/December 2015
(Semester Scheme) (O.S.) (Prior to 2012-13)
MATHEMATICS – III

Time : 3 Hours

Max. Marks : 90

Instruction : Answer *all* questions.

(15×2=30)

I. Answer **any fifteen** of the following :

- 1) Define order of an element.
- 2) Define cyclic group of a group G.
- 3) Find the number of generators of a cyclic group of order 24.
- 4) Prove that every cyclic group is abelian.
- 5) Find all the right cosets of $H = \{0, 3\}$ in group $\{Z_6, \oplus_6\}$.
- 6) State Fermat's theorem.

7) Find the limit of the sequence $\left\{ \frac{4n+3}{5n+4} \right\}$.

8) Show that the sequence $\left\{ \frac{n}{\sqrt{n^2+1}} \right\}$ is convergent.

9) Show that the sequence $\left\{ \frac{3n+4}{2n+1} \right\}$ is monotonic decreasing.

10) Show that the series $1 + \frac{1}{2} + \frac{1}{3} + \dots$ is divergent.

11) State Cauchy's root test for a series of positive terms.

12) Show that the series

$\frac{1}{2} - \frac{1}{2^2} + \frac{1}{2^3} - \frac{1}{2^4} + \dots$ is convergent.



- 13) Define absolute convergence of an alternating series.
- 14) Find the sum of the series $\frac{1}{2} + \frac{1}{3 \cdot 2^2} + \frac{1}{5 \cdot 2^3} + \dots$
- 15) Find the left hand limit of $f(x) = \frac{x^2 - 9}{x - 3}$ at $x = 3$.
- 16) State Rolle's theorem.
- 17) Verify Lagrange's mean value theorem for the function $f(x) = e^x$ in $[0, 1]$
- 18) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right)$ using L' Hospital rule.
- 19) Calculate a_0 in the Fourier series expansion of $f(x) = x$ in the interval $(-\pi, \pi)$
- 20) Write the half range Fourier cosine series for $f(x)$ over the interval $(0, \pi)$

II. Answer any three of the following.

(3x5=15)

- 1) If 'a' is any element of the group G is of order n, then $a^m = e$, for any integer m, iff n divides m.
- 2) Find the order of each element in $G = \{1, -1, i, -i\}$
- 3) In a cyclic group of order K and 'a' is a generator. Prove that $a^m = a^n$ ($m \neq n$) then $m \equiv n \pmod{k}$.
- 4) If $G = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ under multiplication mod (13) and $H = \{1, 3, 9\}$. Find the distinct left cosets.
- 5) State and prove Lagrange's theorem of a finite group G.

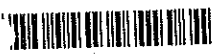
III. Answer any two of the following.

(2x5=10)

- 1) If $\lim_{n \rightarrow \infty} a_n = a$ and $\lim_{n \rightarrow \infty} b_n = b$ then prove that $\lim_{n \rightarrow \infty} (a_n + b_n) = a + b$.

- 2) Test the convergence of the sequence $\left\{ \frac{\log(n+1) - \log(n)}{\sin\left(\frac{1}{n}\right)} \right\}$.

- 3) Prove that a monotonic decreasing sequence bounded below is convergent.



(3x5=15)

IV. Answer any three of the following.

- 1) State and prove D'Alembert's ratio test for the series of positive terms.
- 2) Discuss the convergence of the series

$$\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \dots + \frac{x^n}{(2n-1)(2n+1)} + \dots$$

- 3) Discuss the convergence of the series $\sum \frac{n!}{n^n}$.

- 4) Test the convergence of the series $\sum \left(\frac{n}{n+1}\right)^{n^2}$.

- 5) Sum to infinity the series $\frac{1}{7^1} + \frac{1}{3.7^3} + \frac{1}{5.7^5} + \dots$

V. Answer any two of the following.

(2x5=10)

- 1) Discuss the continuity of f(x) defined by $f(x) = \begin{cases} x^2 - 4 & \text{for } x \neq 2 \\ 5 & \text{for } x = 2 \end{cases}$ at x = 2.

- 2) Verify the Cauchy's mean value theorem for f(x) = x² and g(x) = x³ at (1, 2).

- 3) Expand the function f(x) = log (1 + x) upto the term containing x².

- 4) Evaluate $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$.

VI. Answer any two of the following.

(2x5=10)

- 1) Obtain the Fourier series for the function f(x) = x over (-π, π).

- 2) Obtain the Fourier series for $f(x) = \begin{cases} -x & \text{for } -\pi < x < 0 \\ x & \text{for } 0 < x < \pi \end{cases}$

over the interval (-π, π).

- 3) Find the half range cosine series for the function f(x) = x² over the interval (0, π)

