III Semester B.A./B.Sc. Examination, November/December 2015
(Semester Scheme) (O.S.) (Prior to 2012-13)
MATHEMATICS – III

Time : 3 Hours Max. Marks : 90

***Instruction** : Answer **all** questions.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) Define order of an element.

2) Define cyclic group of a group G.

3) Find the number of generators of a cyclic group of order 24.

4) Prove that every cyclic group is abelian.

5) Find all the right cosets of H = {0, 3} in group $\{Z_6, \oplus_6\}$.

6) State Fermat's theorem.

7) Find the limit of the sequence $\left\{\frac{4n+3}{5n+4}\right\}$.

8) Show that the sequence $\left\{\frac{n}{\sqrt{n^2+1}}\right\}$ is convergent.

9) Show that the sequence $\left\{\frac{3n+4}{2n+1}\right\}$ is monotonic decreasing.

10) Show that the series $1+\frac{1}{2}+\frac{1}{3}+\ldots$ is divergent.

11) State Cauchy's root test for a series of positive terms.

12) Show that the series

$$\frac{1}{2}-\frac{1}{2^2}+\frac{1}{2^3}-\frac{1}{2^4}+\ldots$$ is convergent.

13) Define absolute convergence of an alternating series.

14) Find the sum of the series $\frac{1}{2}+\frac{1}{3.2^2}+\frac{1}{5.2^3}+\ldots$

15) Find the left hand limit of $f(x)=\frac{x^2-9}{x-3}$ at x = 3.

16) State Rolle's theorem.

17) Verify Lagrange's mean value theorem for the function $f(x) = e^x$ in [0, 1]

18) Evaluate $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$ using L′ Hospital rule.

19) Calculate a_0 in the Fourier series expansion of f(x) = x in the interval $(-\pi, \pi)$

20) Write the half range Fourier cosine series for f(x) over the interval $(0, \pi)$

II. Answer **any three** of the following. **(3×5=15)**

1) If 'a' is any element of the group G is of order n, then $a^m = e$, for any integer m, iff n divides m.

2) Find the order of each element in G = {1, –1, i, –i}

3) In a cyclic group of order K and 'a' is a generator. Prove that $a^m = a^n$ $(m \neq n)$ then $m \equiv n \pmod{k}$.

4) If G = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} under multiplication mod (13) and H = {1, 3, 9}. Find the distinct left cosets.

5) State and prove Lagrange's theorem of a finite group G.

III. Answer **any two** of the following. **(2×5=10)**

1) If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ then prove that $\lim_{n\to\infty}(a_n + b_n) = a + b$.

2) Test the convergence of the sequence $\left\{\frac{\log(n+1)-\log(n)}{\sin\left(\frac{1}{n}\right)}\right\}$.

3) Prove that a monotonic decreasing sequence bounded below is convergent.

IV. Answer **any three** of the following. **(3×5=15)**

1) State and prove D'Alembert's ratio test for the series of positive terms.

2) Discuss the convergence of the series

$$\frac{x}{1.3}+\frac{x^2}{3.5}+\frac{x^3}{5.7}+\ldots+\frac{x^n}{(2n-1)(2n+1)}+\ldots$$

3) Discuss the convergence of the series $\sum \frac{n!}{n^n}$.

4) Test the convergence of the series $\sum \left(\frac{n}{n+1}\right)^{n^2}$.

5) Sum to infinity the series $\frac{1}{7^1}+\frac{1}{3.7^3}+\frac{1}{5.7^5}+\ldots$

V. Answer **any two** of the following. **(2×5=10)**

1) Discuss the continuity of f(x) defined by $f(x)=\begin{cases}\frac{x^2-4}{x-2} & \text{for } x\neq 2\\ 5 & \text{for } x=2\end{cases}$ at $x=2$.

2) Verify the Cauchy's mean value theorem for $f(x)=x^2$ and $g(x)=x^3$ at $(1, 2)$.

3) Expand the function $f(x)=\log(1+x)$ upto the term containing x^2.

4) Evaluate $\lim_{x\to 0}(\operatorname{cosec} x-\cot x)$.

VI. Answer **any two** of the following. **(2×5=10)**

1) Obtain the Fourier series for the function $f(x)=x$ over $(-\pi, \pi)$.

2) Obtain the Fouriers series for $f(x)=\begin{cases}-x & \text{for } -\pi<x<0\\ x & \text{for } 0<x<\pi\end{cases}$

over the interval $(-\pi, \pi)$.

3) Find the half range cosine series for the function $f(x)=x^2$ over the interval $(0, \pi)$
